AN ENGINEERING METHOD OF CALCULATING CONVECTIVE HEAT TRANSFER FOR A BODY IN AN ATTACHED GAS FLOW

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An engineering method is proposed for calculating the distribution of local heat transfer coefficients in longitudinal and cross flow over a plate and cross flow over a cylinder and sphere.

Heat transfer in gas flow over a body plays a major role in a number of areas of technology and to a considerable extent determines the safety and durability of building elements.

There is as yet, however, no simple and reliable engineering method of calculating heat transfer for attached gas flow over a body.

Existing analytic methods of calculation are laborious and valid only for particular cases.

The present paper aims at giving a simple and reliable method of calculating laminar and turbulent thermal boundary layers.

The equation of an axisymmetric thermal boundary layer

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2} \tag{1}$$

with the help of the equation of continuity

$$\frac{\partial (uD)}{\partial x} + \frac{\partial (vD)}{\partial y} = 0 \tag{2}$$

for the boundary conditions

$$\begin{gathered} u = v = 0, \quad T = T_{\mathrm{W}} \ \text{when} \ y = 0, \\ u = u_0, \quad T = T_0 \ \text{when} \ y = \delta \end{gathered} \tag{3}$$

may be represented as the following integral relation [1]:

$$\frac{d\,\mathrm{Pe}_\theta}{d\bar{x}} + \mathrm{Pe}_\theta\left(\frac{1}{D}\frac{dD}{d\bar{x}} + \frac{1}{t_0}\frac{dt_0}{d\bar{x}}\right) = \mathrm{St}\,\mathrm{Pe}_L, \tag{4}$$

where $\mathrm{Pe}_\theta = \dfrac{u_0\theta}{a_0}$; $\bar{x} = \dfrac{x}{L}$;

$$\mathrm{St} = \frac{q_{\mathrm{W}}}{\rho_0 u_0 c_p g t_0}; \quad \theta = \int\limits_0^\delta \frac{\rho u}{\rho_0 u_0}\left(1 - \frac{T^* - T_{\mathrm{W}}}{T_{00} - T_{\mathrm{W}}}\right) dy;$$

$$t_0 = T_{\mathrm{W}} - T_{00}; \quad \mathrm{Pe}_L = \frac{u_0 L}{a_0};$$

$$q_{\mathrm{W}} = -\lambda \left.\frac{dT}{dy}\right|_{y=0}.$$

The integral relation (4) is valid both for laminar and turbulent boundary layers. It is a modification of the equation of the thermal boundary layer (1) and should therefore be called the equation of the thermal boundary layer.

To solve the thermal boundary layer (4) we must know the connection between the parameter Pe_θ, based on the characteristic boundary layer thickness θ, and the St number, based on the parameters at the edge of the boundary layer.

It has been shown [1, 2] that the relation between the local Pe_θ number and St is conservative with respect to the pressure gradient and variations in wall temperature.

To establish this connection we shall use solutions of the laminar and turbulent boundary layers for a plate with constant wall temperature.

The equation of the thermal boundary layer, both laminar and turbulent, for a plate with constant wall temperature has the form

$$\frac{d\,\mathrm{Pe}_\theta}{d\bar{x}} = \mathrm{St}\,\mathrm{Pe}_L \quad \text{or} \quad \frac{d\,\mathrm{Pe}_\theta}{d\,\mathrm{Pe}_x} = \mathrm{St}; \tag{5}$$

with the boundary condition: when $\bar{x} = 0$ $\mathrm{Pe}_\theta \approx 0$.

To establish the connection between Pe_θ and St in a laminar boundary layer, we shall use the heat transfer equation for a flat isothermal plate [8]

$$\mathrm{Nu}_x = 0.332\sqrt{\mathrm{Pe}_x}\sqrt[3]{\mathrm{Pr}}. \tag{6}$$

Equation (6) may be put in the following form:

$$\mathrm{St} = 0.332\,\mathrm{Pe}_x^{-\frac{1}{2}}\,\mathrm{Pr}^{-\frac{1}{6}}. \tag{7}$$

Substituting the value of the St number from (7) in (5) and integrating, we find the following connection between Pe_θ and Pe_x for a laminar boundary layer:

$$\mathrm{Pe}_\theta = 0.664\sqrt{\mathrm{Pe}_x}\,\mathrm{Pr}^{-\frac{1}{6}}. \tag{8}$$

Substituting Pe_θ for Pe_x in (7), we obtain the relation between the St number and the Pe number, based on the characteristic boundary layer thickness θ, for the laminar region:

$$\mathrm{St} = 0.22\,\mathrm{Pe}_\theta^{-1}\,\mathrm{Pr}^{-\frac{1}{3}}. \tag{9}$$

To establish the relation between the Pe_θ and St numbers in a turbulent boundary layer, we shall use the equation of turbulent heat transfer for a flat isothermal plate [3]

$$\mathrm{Nu}_x = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.4} \tag{10}$$

or

$$\mathrm{St} = 0.0296\,\mathrm{Re}_x^{-0.2}\,\mathrm{Pr}^{-0.4}. \tag{10}$$

As for a laminar boundary layer, we now establish the connection between Pe_θ and Pe_x for a turbulent boundary layer, using (5) and (10):

$$\mathrm{Pe}_\theta = 0.037\,\mathrm{Pe}_x^{0.8}\,\mathrm{Pr}^{-0.4}. \tag{11}$$

Substituting the value of Pe_x obtained from (11) in (10), we get a relation between St and Pe_θ, based on the characteristic boundary layer thickness θ, for the turbulent region:

$$\mathrm{St} = 0.013\,\mathrm{Pe}_\theta^{-0.25}\,\mathrm{Pr}^{-0.5}. \tag{12}$$

In practical calculations the influence of the temperature factor on the relation between the St and Pe_θ numbers in the laminar boundary layer may be neglected [4].

For a turbulent boundary layer, it may be assumed that the heat transfer law (12) depends on the temperature factor as follows [1]:

$$\left.\frac{\mathrm{St}}{\mathrm{St}_0}\right|_{\mathrm{Pe}_\theta=\mathrm{const}} = \bar{T}_W^{-0.5}, \tag{13}$$

which is valid in the important practical region of values of $\bar{T}_W = T_W/T_{00}$ from 0.5 to 3.

Thus, the heat transfer law for a turbulent boundary layer will have the form:

$$\mathrm{St} = 0.013\,\mathrm{Pe}_\theta^{-0.25}\,(\bar{T}_W\,\mathrm{Pr})^{-0.5}. \tag{14}$$

In calculating the heat transfer in flow over bodies it is necessary to determine the distribution of specific heat flux over the body length, knowing its geometry, the law of variation of the wall temperature T_W, and the parameters of the stream (u_∞, p_∞, T_∞).

Let us consider the motion of a gas with uniform distribution of velocity and temperature in the stream. The motion of the gas inside the boundary layer will be assumed isentropic, with geometrical effects determined by the geometry of the body.

From the geometry and the perfect fluid solution we determine the variation of velocity at the edge of the boundary layer $u_0 = u_\infty f(\bar{x})$ and the local Peclet number $Pe_L = Pe_{L_0} f(\bar{x})$.

The equation of the thermal boundary layer (4) may be written in the form:

$$\frac{d\,\mathrm{Pe}_\theta}{d\bar{x}} + \mathrm{Pe}_\theta \frac{d}{d\bar{x}} \ln[\bar{D}(\bar{T}_w - 1)] = \mathrm{St}\,\mathrm{Pe}_L \tag{15}$$

or for a laminar boundary, taking into account (9) and (14):

$$\frac{d\,\mathrm{Pe}_\theta}{d\bar{x}} + \mathrm{Pe}_\theta \frac{d}{d\bar{x}} \ln[\bar{D}(\bar{T}_w - 1)] = \frac{0.22\,\mathrm{Pe}_{L_0} f(\bar{x})}{\mathrm{Pe}_\theta \sqrt[3]{\mathrm{Pr}}}, \tag{16}$$

with the boundary condition: when $\bar{x} = 0$ $Pe_\theta = 0$, $\bar{D} = 0$, $\bar{T}_w = \bar{T}_{w_1}$; and for a turbulent boundary layer:

$$\frac{d\,\mathrm{Pe}_\theta}{d\bar{x}} + \mathrm{Pe}_\theta \frac{d}{d\bar{x}} \ln[\bar{D}(\bar{T}_w - 1)] = \frac{0.013\,\mathrm{Pe}_{L_0} f(\bar{x})}{\mathrm{Pe}_\theta^{0.25}(\mathrm{Pr}\,\bar{T}_w)^{0.5}}, \tag{17}$$

with boundary condition: when $\bar{x} = \bar{x}_{cr}$ $Pe_\theta = Pe_{\theta_{cr}}$, $\bar{D} = \bar{D}_1$, $\bar{T}_w = \bar{T}_{w_1}$.

On solving differential equations (16) and (17), we obtain expressions for the laminar and turbulent boundary layers:

$$\mathrm{Pe}_\theta = \frac{1}{\bar{D}(\bar{T}_w - 1)}\left(0.22\mathrm{Pe}_{L_0}\,\mathrm{Pr}^{-\frac{1}{2}} \int_0^{\bar{x}} f(\bar{x})[\bar{D}(\bar{T}_w - 1)]^2 d\bar{x}\right)^{\frac{1}{2}}, \tag{18}$$

$$\mathrm{Pe}_\theta = \frac{\bar{D}_1(\bar{T}_{w_1} - 1)}{\bar{D}(\bar{T}_w - 1)}\left\{\mathrm{Pe}_{\theta_{cr}}^{1.25} + \int_{\bar{x}_{cr}}^{\bar{x}} \frac{0.01625 f(\bar{x})}{(\bar{T}_w\,\mathrm{Pr})^{0.5}} \times \left[\frac{\bar{D}(\bar{T}_w - 1)}{\bar{D}_1(\bar{T}_{w_1} - 1)}\right]^{1.25} \mathrm{Pe}_{L_0}\,d\bar{x}\right\}^{0.8}. \tag{19}$$

Using the laminar heat transfer law (9) and relation (18), we obtain an equation for calculating the heat transfer coefficient of the laminar boundary layer in a subsonic gas flow over a body:

$$\mathrm{Nu}_L = \frac{0.332\bar{D}(\bar{T}_w - 1)\sqrt{\mathrm{Pe}_{L_0}} f(\bar{x})}{\left(\int_0^{\bar{x}} f(\bar{x})[\bar{D}(\bar{T}_w - 1)]^2 d\bar{x}\right)^{\frac{1}{2}} \mathrm{Pr}^{\frac{1}{6}}}. \tag{20}$$

Similarly, (14) and (19) can be used to compute the heat transfer coefficient for a turbulent boundary layer with subsonic flowpast by a gas:

$$\mathrm{Nu}_L = \left\{0.0296\,\mathrm{Pe}_{L_0}^{0.8} f(\bar{x})[\bar{D}(\bar{T}_w - 1)]^{0.25}(\mathrm{Pr}\,\bar{T}_w)^{-0.4}\right\}\left\{[\mathrm{Pe}_{\theta_{cr}}^{1.25}(\mathrm{Pr}\bar{T}_w)^{0.5}[\bar{D}_1(\bar{T}_{w_1} - 1)]^{1.25}/0.01625\mathrm{Pe}_{L_0} + \int_{x_{cr}}^{\bar{x}} f(\bar{x})[\bar{D}(\bar{T}_w - 1)]^{1.25} d\bar{x}]^{0.2}\right\}^{-1}, \tag{21}$$

where $\bar{x}_{cr}$ is the point of transition between a laminar boundary layer and turbulence, and L is a characteristic dimension.

According to (20) and (21), the variation of the heat transfer coefficient over the length of the body may easily be calculated, if the variation of the wall temperature and the velocity at the edge of the boundary layer u_0 are known.

The equation of heat transfer for longitudinal laminar flow over a flat plate with wall temperature varying according to the law $\bar{T}_w = \bar{T}_{w_1} + A\bar{x}$ has the form:

$$\mathrm{Nu}_x = 0.332\sqrt{\mathrm{Re}_x}\sqrt[3]{\mathrm{Pr}}\left(\frac{3A\bar{x}(\bar{T}_{w_1} - 1 + A\bar{x})}{(\bar{T}_{w_1} - 1 + A\bar{x})^3 - (\bar{T}_{w_1} - 1)^3}\right)^{\frac{1}{2}}, \tag{22}$$

and for a turbulent boundary layer:

$$\mathrm{Nu}_x = \left[0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{0.4}\,(\bar{T}_{w_1} - 1 + A\bar{x})^{0.25}\right]\left[\{\mathrm{Re}_{x_{cr}}/\mathrm{Re}_x + [(\bar{T}_{w_1} - 1 + A\bar{x})^{2.25} - (\bar{T}_{w_1} - 1 + A\bar{x}_{cr})^{2.25}]/2.25A\bar{x}\}^{0.2}\,\bar{T}_w^{0.4}\right]^{-1}. \tag{23}$$

For cross flow over a flat plate of infinite length and width L, the velocity varies according to (5):

$$f(\bar{x}) = \frac{u_0}{u_\infty} = \frac{\bar{x}}{\sqrt{1-\bar{x}^2}}, \qquad \text{where } \bar{x} = \frac{x}{L/2}; \tag{24}$$

for a cylinder this has the form:

$$f(\bar{x}) = \frac{u_0}{u_\infty} = 2\sin 2\bar{x}, \qquad \text{where } \bar{x} = \frac{x}{D_0}; \tag{25}$$

and for a sphere:

$$f(\bar{x}) = \frac{u_0}{u_\infty} = \frac{3}{2}\sin 2\bar{x}. \tag{26}$$

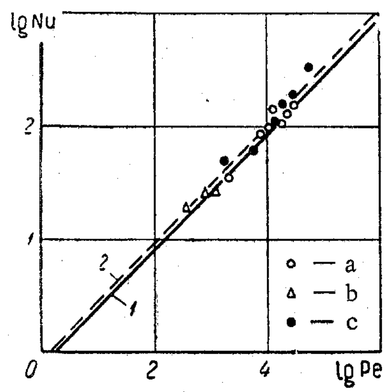

Fig. Comparison of theoretical and experimental heat transfer data at the stagnation point of a cylinder: 1 – according to Eq. (30), 2 – according to [8], a, b, c – according to [6], [7], and [8], respectively.

Accordingly, the equation of heat transfer for cross flow over an isothermal flat plate with a laminar boundary layer has the form:

$$\mathrm{Nu}_L = 0.47\sqrt{\mathrm{Re}_{L_0}}\,\sqrt[3]{\mathrm{Pr}}\,\bar{x}\left[\left(1-\sqrt{1-\bar{x}^2}\right)(1-\bar{x}^2)\right]^{-\frac{1}{2}}. \tag{27}$$

Accordingly, the heat transfer equation for a transversely flowed past cylinder at a constant temperature can be represented in the form:

$$\mathrm{Nu}_D = 0.664\sqrt{\mathrm{Re}_D}\,\sqrt[3]{\mathrm{Pr}}\,\frac{\sin 2\bar{x}}{(1-\cos 2\bar{x})^{1/2}}. \tag{28}$$

For a sphere:

$$\mathrm{Nu}_D = 0.7\sqrt{\mathrm{Re}_D}\,\sqrt[3]{\mathrm{Pr}}\,\frac{\sin^2 2\bar{x}}{\left(\frac{1}{2}\cos^3 2\bar{x} - \frac{3}{2}\cos 2\bar{x} + 1\right)^{\frac{1}{2}}}. \tag{29}$$

For the stagnation point, (28) takes the form:

$$\mathrm{Nu}_D = 0.94\sqrt{\mathrm{Re}_D}\,\sqrt[3]{\mathrm{Pr}}, \quad \text{where } \mathrm{Re}_D = \frac{\rho_\infty u_\infty D_0}{\mu_{00}}, \tag{30}$$

since

$$\lim_{\bar{x}\to 0}\frac{\sin 2\bar{x}}{(1-\cos 2\bar{x})^{1/2}} = \frac{2}{\sqrt{2}}. \tag{31}$$

It can be seen from the figure that the calculations based on (30) are in satisfactory agreement with the data of other authors.

NOTATION

u and v – longitudinal and transverse velocity components in boundary layer; T – temperature in boundary layer; x – coordinate in direction of washed surface; y – coordinate normal to washed surface; T_W – wall temperature; δ – boundary layer thickness; T^* – stagnation temperature at boundary layer; T_{00} – stagnation temperature inside boundary layer; T_∞ – free stream temperature; u_∞ – free stream velocity; D_0 – characteristic diameter of body; D – local diameter of an axisymmetric body; c_p – specific heat capacity at constant pressure; ρ_0 – density at edge of boundary layer; u_0 – velocity at edge of boundary layer; a_0 – thermal diffusivity at stagnation temperature at edge of boundary layer; g – acceleration of gravity; ρ – density in boundary layer; μ_0 – coefficient of dynamic viscosity at temperature at edge of boundary layer; λ – thermal conductivity; Pr – Prandtl number; θ – energy loss thickness; Pe_θ – Peclet number based on energy loss thickness; Re_x – Reynolds number based on x; Pe_{Lo} – Peclet number based on parameters of undisturbed flow; Re_D – Reynolds number based on parameters of undisturbed flow.

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